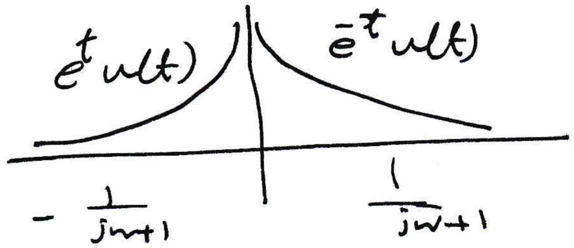
If we interchange t and w, we obtain

$$2\pi f(-w) = \int_{-\infty}^{\infty} F(t)\, e^{-jwt}\, dt = \mathcal{F}[F(t)]$$

<u>EX</u> $f(t) = e^{-|t|}$ then

$$F(w) = \frac{2}{w^2+1}$$



⑧ Time integration

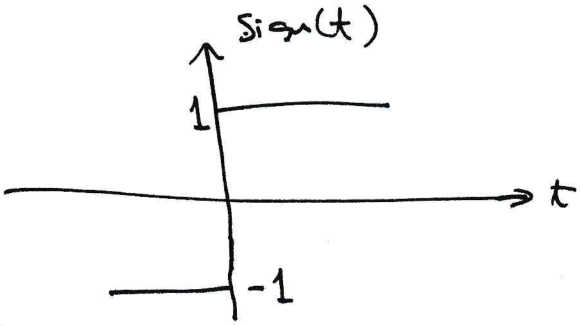
$F(w) = \mathcal{F}[f(t)]$ then

$$\mathcal{F}\left[\int_{-\infty}^{\infty} f(t)\, dt\right] = \frac{F(w)}{jw} + \pi\, \delta(w).$$

$f(t)$	$F(w)$
$\delta(t)$	1
1	$2\pi\,\delta(w)$
$u(t)$	$\pi\,\delta(w) + \frac{1}{jw}$
$u(t+T)-u(t-T)$	$2\,\frac{\sin wT}{w}$
$\lvert t\rvert$	$-2/w^2$
$\mathrm{sgn}(t)$	$2/jw$
$e^{-at}u(t)$	$\frac{1}{jw+a}$
$e^{at}u(t)$	$\frac{1}{jw-a}$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+jw)^{n+1}}$
$e^{-a\lvert t\rvert}$	$2a/(a^2+w^2)$
e^{jw_0t}	$2\pi\,\delta(w-w_0)$
$\sin w_0t$	$j\pi\,[\delta(w+w_0)-\delta(w-w_0)]$
$\cos w_0t$	$\pi\,[\delta(w+w_0)+\delta(w-w_0)]$
$e^{-at}\sin w_0t\,u(t)$	$\frac{w_0}{(a+jw)^2+w_0^2}$
$e^{-at}\cos w_0t\,u(t)$	$\frac{a+jw}{(a+jw)^2+w_0^2}$

EX Find the Fourier transform of the following function

a) Sighn function b) double-sided exponential e^{-at}.



a) ① first way

$f(t) = Sgn(t) = -1 + 2u(t)$

take the F.T of each terms gives:-

$F(w) = -2\pi\delta(w) + 2(\pi\delta(w) + \frac{1}{jw})$

$= \frac{2}{jw}$

$= u(t) - u(-t)$

$= u(t) - (1 - u(t))$

$= -1 + 2u(t)$

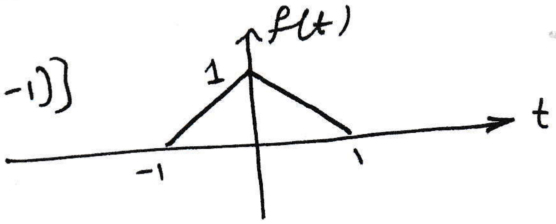
② second way

$f'(t) = 2\delta(t)$

take the F.T. $\Rightarrow jw F(w) = 2 \Rightarrow F(w) = \frac{2}{jw}$

b) $f(t) = e^{-a|t|} = \frac{2a}{a^2 + w^2}$

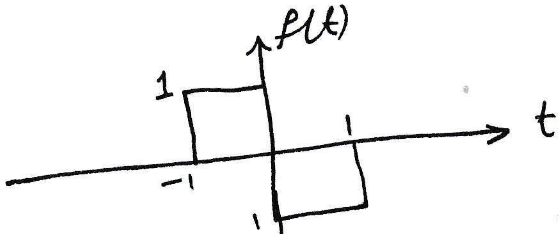
EX Find the F.T of the figure below?



$-w^2 F(w) = F(\delta(t+1) - 2\delta(t) + \delta(t-1))$

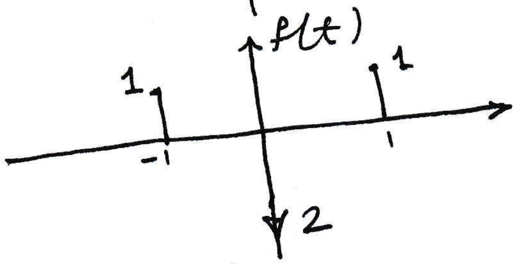
$-w^2 F(w) = e^{jw} - 2 + e^{-jw}$

$\Rightarrow F(w) = \frac{e^{jw} + e^{-jw} - 2}{-w^2}$



$F(w) = \frac{\cos w - 2}{-w^2}$

$= \frac{2 - \cos w}{w^2}$



Or by mathematically method:-

$$f(t) = \begin{cases} 1-t & 0 < t < 1 \\ 1+t & -1 < t < 0 \end{cases}$$

$$\therefore F(w) = \int_0^1 (1-t)\, e^{-jwt}\, dt + \int_{-1}^{0} (1+t)\, e^{-jwt}$$

$$= \int_0^1 e^{-jwt} - \int_0^1 t\, e^{-jwt}\, dt + \int_{-1}^{0} e^{-jwt} + \int_0^1 t\, e^{-jwt}\, dt$$

$$= \int_0^1 e^{-jwt} - \int_0^1 t\, e^{-jwt}\, dt + \int_{-1}^{0} e^{-jwt} + \int_0^1 t\, e^{-jwt}\, dt$$

dif fn	int fn
t +	e^{-jwt}
1 −	$-\frac{e^{-jwt}}{jw}$
	$-\frac{e^{-jwt}}{w^2}$

$$= -\frac{1}{jw} e^{-jwt}\Big|_0^1 + \frac{t}{jw} e^{-jwt}\Big|_0^1 - \frac{e^{-jwt}}{w^2}\Big|_0^1$$

$$-\frac{1}{jw} e^{-jwt}\Big|_{-1}^{0} - \frac{t}{jw} e^{-jwt}\Big|_{-1}^{0} + \frac{e^{-jwt}}{w^2}\Big|_{-1}^{0}$$

$$= -\frac{1}{jw} e^{jw} + \frac{1}{w^2} - \frac{1}{w^2} e^{jw} - \frac{1}{jw} + \frac{e^{jw}}{jw} - \frac{e^{-jw}}{jw} + \frac{1}{jw} - \frac{e^{-jw}}{w^2}$$

$$+ \frac{1}{w^2} + \frac{e^{-jw}}{jw} = \frac{2}{w^2} - \frac{(e^{jw} + e^{-jw})}{w^2} = \frac{2 - \cos w}{w^2}$$

EX obtain the inverse Fourier transform of:-

a) $F(w) = \dfrac{10jw+4}{(jw)^2 + 6jw + 8}$ b) $G(w) = \dfrac{w^2+21}{w^2+9}$

ⓐ by replacing $jw \rightarrow s$

$$F(s) = \frac{10s+4}{s^2+6s+8} = \frac{10s+4}{(s+4)(s+2)} = \frac{A}{s+4} + \frac{B}{s+2}$$

where $A = \left.\frac{10s+4}{s+2}\right|_{s=-4} = \left.\frac{10s+4}{s+2}\right|_{s=-4} = 18$

$B = (s+2)\,F(s)\Big|_{s=-2} = \left.\frac{10s+4}{s+4}\right|_{s=-2} = -8$

$\therefore F(s) = \frac{18}{s+4} - \frac{8}{s+2}$

$F(jw) = \frac{18}{jw+4} + \frac{-8}{jw+2} \Rightarrow f(t) = (18e^{-4t} - 8e^{-2t})\,u(t)$

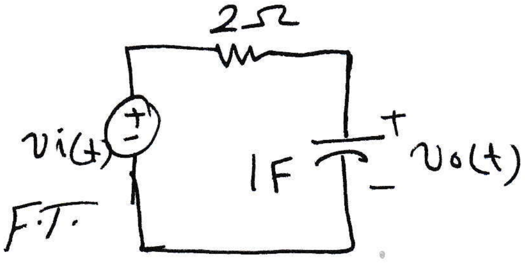
b) $G(w) = \frac{w^2+21}{w^2+9}$

$= \frac{w^2+9+12}{w^2+9} = \frac{w^2+9}{w^2+9} + \frac{12}{w^2+9}$

$= 1 + \frac{12}{w^2+9}$

$g(t) = \delta(t) + 4\,e^{-3|t|}$

Circuit application :-



EX Find $v_o(t)$ in the cct for $v_i(t) = 2e^{-3t}u(t)$ ~~by F.T.~~ by F.T.

F.T of the input voltage

$V_i(w) = \frac{2}{3+jw}$

and the transfer function obtain by voltage division is

$$H(w) = \frac{V_o(w)}{V_i(w)} = \frac{\frac{1}{jw}}{2 + \frac{1}{jw}} = \frac{1}{1+j2w}$$

$$V_o(w) = V_i(w)\,H(w) = \frac{2}{(3+jw)(1+j2w)}$$

or

$$V_o(w) = \frac{\cancel{2}}{(3+jw)(0.5+jw)\cancel{2}}$$

By partial fraction

$$V_o(s) = \frac{1}{(s+3)(s+0.5)} = \frac{A}{s+3} + \frac{B}{s+0.5}$$

$$A = \frac{1}{(s+3)(s+0.5)} * (s+3)\Big|_{s=-3} = -0.4$$

$$B = \frac{1}{(s+3)(s+0.5)} * (s+0.5)\Big|_{s=-0.5} = 0.4$$

$$V_o(w) = \frac{-0.4}{3+jw} + \frac{0.4}{0.5+jw}$$

Taking the inverse F.T

$$\therefore V_o(t) = -0.4\,e^{-3t}u(t) + 0.4\,e^{-0.5t}u(t)$$

Parsevals theorem :-

$$W = \int_{-\infty}^{\infty} P(t)\, dt$$

$P(t) \longrightarrow$ power

$W \longrightarrow$ energy

where $P(t) = v^2(t) = i^2(t) = f^2(t)$ at $R = 1\Omega$

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)\, dt$$

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(w)|^2\, dw$$ ← parsevals theorem

Proof

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)\, dt = \int_{-\infty}^{\infty} f(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)\, e^{jwt}\, dw\right] dt$$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\, F(w)\, e^{jwt}\, dw\, dt$$

Reversing the order of integration:-

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)\left[\int_{-\infty}^{\infty} f(t)\, e^{-j(-w)t}\, dt\right] dw$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)\, F(-w)\, dw = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)\, F(w)^{*}\, dw$$

$$\Rightarrow W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(w)^2|\, dw$$

EX The voltage across a $10\,\Omega$ resistor is $v(t) = 5e^{-3t}u(t)$. Find the total energy dissipated in the resistance.

$f(t) = v(t)$ or $F(w) = V(w)$

In the frequency domain

$$F(w) = V(w) = \frac{5}{3+jw}$$

So that $|F(w)|^2 = F(w) * F(w)^* = \frac{25}{9+w^2}$

that energy dissipated is :-

$$W_{10\Omega} = \frac{10}{2\pi}\int_{-\infty}^{\infty} |F(w)|^2\, dw = \frac{10}{\pi}\int_{0}^{\infty} \frac{25}{9+w^2}\, dw$$

$$= \frac{-250}{\pi}\left(\frac{1}{3}\tan^{-1}\frac{w}{3}\right)_0^{\infty} = \frac{250}{\pi}\left(\frac{1}{3}\right)\left(\frac{\pi}{2}\right)$$

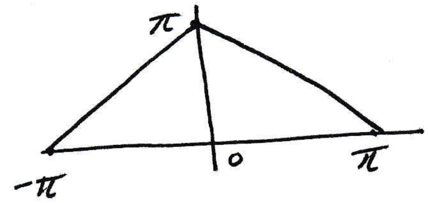
$$= \frac{250}{6} = 41.67\ J.$$

Fourier Series

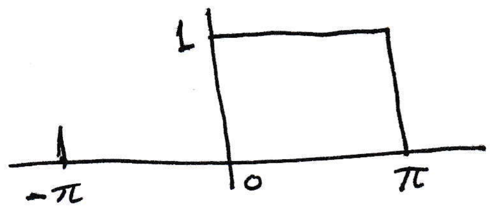
Homework 1

Ⓐ Find the Fourier Series for figure below.

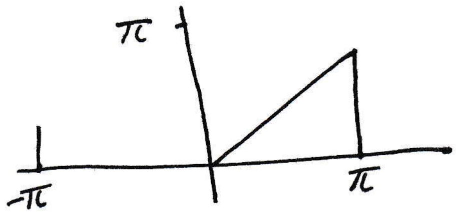
a)



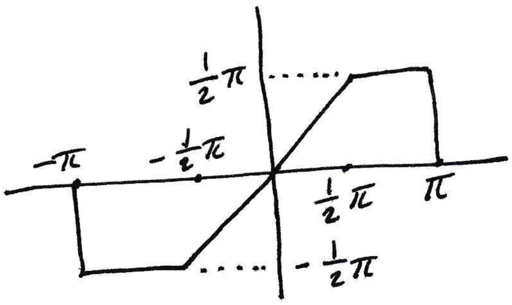
b)



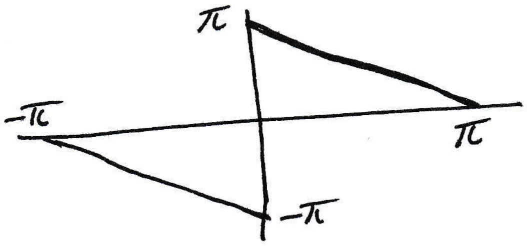
c)



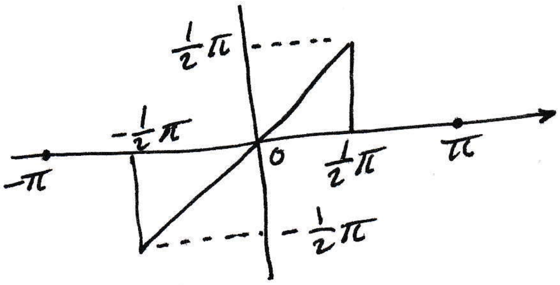
d)



e)



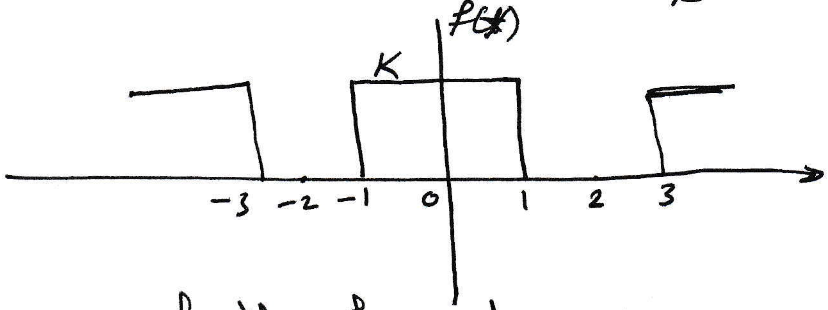
f)



Fourier Series

Home work 2

Q1 Find the Fourier series of the function as in figure below.

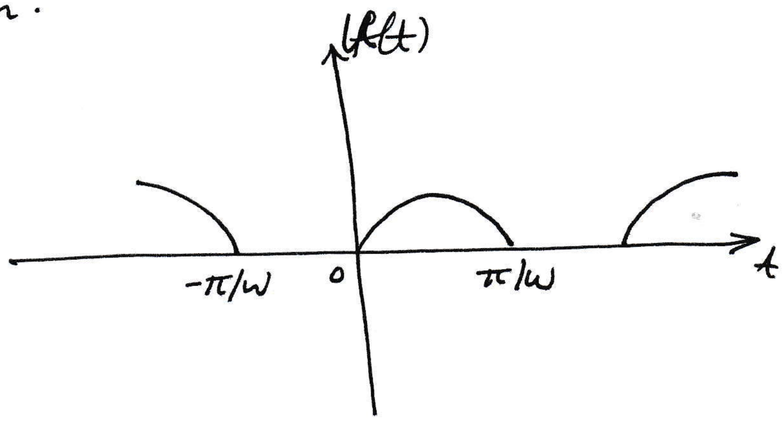


Q2 Find the Fourier Series of the function.

$$f(x) = \begin{cases} -K & \text{if } -2 < x < 0 \\ K & \text{if } 0 < x < 2 \end{cases}$$

Q3 A sinusoidal voltage $E \sin \omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting periodic function.

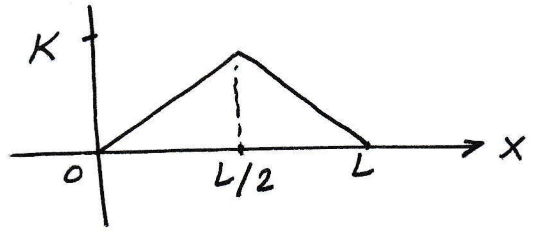
$$u(t) = \begin{cases} 0 & \text{if } -L < t \leq 0 \\ E \sin \omega t & \text{if } 0 < t \leq L \end{cases}$$



Q4 Find the Fourier Series of the function

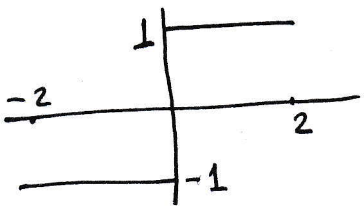
$$f(x) = x + \pi \quad \text{if } -\pi < x \leq \pi$$

Q5 Find the two half-range expansion of the figure below?

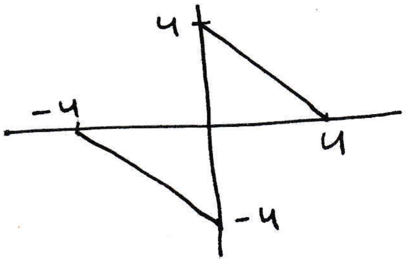


Q6 Find all functions that are both even and odd.

a)



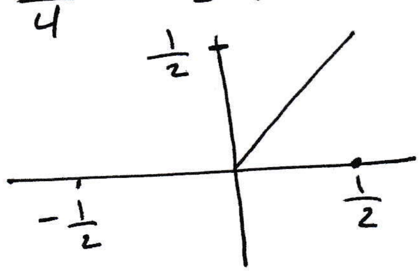
b)



c) $f(x) = x^2 \quad -1 < x < 1$

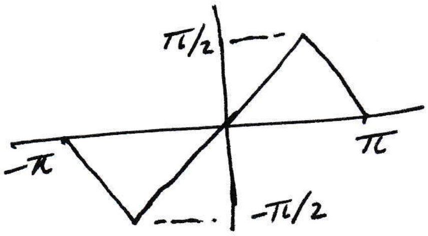
d) $f(x) = 1 - \frac{x^2}{4} \quad -2 < x < 2$

e)

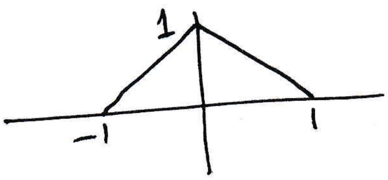


f) $f(x) = \cos \pi x \quad -\frac{1}{2} < x < \frac{1}{2}$

g)



10)



Fourier Series — Homework 3

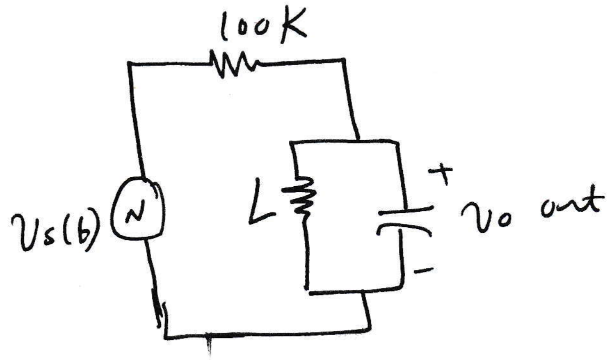
Q1 For Figure below $v_s(t) = 10$ volt $\quad 0 < t < \pi$ msec

$= 0$ volt $\quad \pi < t < 2\pi$ sec

Determine the value of $v_o(t)$ at

① $L = 1\,H$, $C = 1\,\mu F$

② $L = 1/9\,H$, $C = 1\,\mu F$



Q2 the voltage across the terminal for circuit is

$v(t) = 30 + 20\cos(120\pi t + 45) + 10\cos(120\pi - 45)\ v.$

the current entering the terminal at higher potential

is $i(t) = 6 + 4\cos(120\pi + 10) - 2\cos(120\pi t - 60)\ t.$

Find

1) the R.M.S value of the voltage

② the R.M.S value of current.

3) the average value of the power observed by the circuit.

Fourier Series — Homework 4

Q1: show that the integral represents the indicated function and by using $f(x) = \int_0^\infty [A(w)\cos wx + B(w)\sin wx]\, dw$ — Fourier Cosine integral and Fourier Cosine integral for these functions.

1. $\int_0^\infty \frac{\cos xw + w \sin xw}{1+w^2}\, dx = \begin{cases} 0 & \text{if } x<0 \\ \pi/2 & \text{if } x=0 \\ \pi e^{-x} & \text{if } x>0 \end{cases}$

2- $\int_0^\infty \frac{\sin \pi w \sin xw}{1-w^2}\, dw = \begin{cases} \frac{\pi}{2}\sin x & \text{if } 0 \le x \le \pi \\ 0 & \text{if } x>\pi \end{cases}$

3- $\int_0^\infty \frac{1-\cos \pi w}{w} \sin xw\, dw = \begin{cases} \frac{1}{2}\pi & \text{if } 0<x<\pi \\ 0 & \text{if } x>\pi \end{cases}$

4- $\int_0^\infty \frac{\sin w - w\cos w}{w^2} \sin xw\, dw = \begin{cases} \frac{1}{2}\pi x & \text{if } 0<x<1 \\ \frac{1}{4}\pi & \text{if } x=1 \\ 0 & \text{if } x>1 \end{cases}$

Q2 By using Fourier Cosine integral representation.

1- $f(x) = \begin{cases} 1 & \text{if } 0<x<1 \\ 0 & \text{if } x>1 \end{cases}$

2- $f(x) = \begin{cases} x^2 & \text{if } 0<x<1 \\ 0 & \text{if } x>1 \end{cases}$

3- $f(x) = \begin{cases} \sin x & \text{if } 0<x<\pi \\ 0 & \text{if } x>\pi \end{cases}$

Q3 By using Fourier Sine integral representations.

1) $f(x) = \begin{cases} x & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$

2) $f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases}$

3) $f(x) = \begin{cases} \cos x & \text{if } 0 < x < \pi \\ 0 & \text{if } x > \pi \end{cases}$

4) $f(x) = \begin{cases} e^{-x} & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases}$

Fourier Series Homework 5

1- Find the cosine transform of

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ -1 & 1 < x < 2 \\ 0 & x > 2 \end{cases}$$

2. Find the cosine transform of

$$f(x) = \begin{cases} x & 0 < x < 2 \\ 0 & x > 2 \end{cases}$$

3- find the cosine transform of

$$f(x) = \begin{cases} x^2 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$$

4- find $F_s(e^{-ax}),\ a > 0$

5 - Find the Sine transform of

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ -1 & 1 < x < 2 \\ 0 & x > 2 \end{cases}$$

Fourier Series — Homework 6

1- Find the Fourier transform of $f(x)$

ⓐ $f(x) = \begin{cases} e^{2ix} & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

b) $f(x) = \begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$

c) $f(x) = \begin{cases} x & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$